Doppler-stabilized fiber link with 6 dB noise improvement below the classical limit


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It is known that temperature variations and acoustic noise affect ultrastable frequency dissemination along optical fiber. Active stabilization techniques are adopted to compensate for the fiber-induced phase noise. However, despite this compensation, the ultimate link performances are limited by the delay-unsuppressed noise that is related to the propagation delay of the light in the fiber. We demonstrate a post-processing approach which enables us to overcome this limit. We implement a subtraction algorithm between the optical signal delivered at the remote link end and the round-trip signal. In this way, a 6 dB improvement beyond the delay-unsuppressed noise is obtained. We confirm the prediction with experimental data obtained on a 47 km metropolitan fiber link and propose how to extend this method for frequency dissemination.

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In the last decade, optical fiber links for frequency dissemination have become a key tool in frequency metrology. The coherent transfer of optical [1–5] and radio-frequency signals [6–9] through optical fibers encompasses the present satellite techniques in terms of resolution, enabling a number of significant applications, such as the remote comparison of atomic clocks at the $10^{-18}$ uncertainty level [10–12] and the investigation of new frontiers in fundamental physics, geodesy [13], and radioastronomy [14].

Coherent frequency links through optical fiber are based on the transfer of an ultrastable laser frequency signal along a standard telecom fiber. It is well known that length variations because of temperature and mechanical stresses affect the fiber, resulting in a severe deterioration of the delivered signal phase stability. To overcome this difficulty, it is a common practice to adopt a Doppler noise cancellation scheme in which a double pass of the light in the fiber in opposite directions is exploited [15]. Basically, from the link remote end, a part of the radiation is reflected back to the local laboratory and here phase-compared to the original signal. Their beat note allows the detection of the fiber phase noise, which is actively cancelled through a phase-locked loop. Hence, at the remote end, the beat note between the delivered laser and the local frequency reference is in principle not affected by the fiber phase noise [3]. Actually, the round-trip delay imposes an ultimate limit to the link noise suppression. In fact, the phase noise detection and cancellation are accomplished on the round-trip signal, whereas the delivered signal travels the fiber in a single pass and suffers from a residual noise because of the fiber delay. Even for a perfect Doppler cancellation at the local laboratory, the noise reduction factor on the delivered signal is $\frac{1}{2}(2\pi f_{r}c)^2$ for Fourier frequencies $f \ll \frac{1}{c}$, where $c$ is the one-way propagation time. [3].

There are two main applications of optical links: either the signal is directly used as an optical frequency reference or, in most of the cases, it is post-processed to perform, for instance, clocks comparisons and spectroscopy.

In this Letter, we describe a novel technique that implements the real-time frequency dissemination at the level of existing Doppler-stabilized links and enables the post-processed dissemination at an improved level of stability. We also report on the experimental results we obtained on a 47 km long optical link deployed in the metropolitan area and used for data transmission.

As in the Doppler cancellation technique, we intend to compensate the phase of the optical signal delivered at the remote link end, $\phi_{r}(t)$, by detecting the phase of the beat note between the original and the round-trip radiation, $\phi_{rt}(t)$. As a first step, we evaluate the relation between the fiber phase noise and the phase noise of the signal transmitted at the remote link end in terms of their time evolution, following the approach described in [3]. If $\delta \phi(z, t)dz$ is the phase noise on the fiber at time $t$ and position $z$, $\phi_{r}(t)$ can be written as

$$\phi_{r}(t) = \int_{0}^{L} \delta \phi\left(z, t - \tau + \frac{z}{c_{n}}\right)dz, \tag{1}$$

where $L$ is the link length, $c_{n}$ is the speed of light in the fiber, and $\tau = L/c_{n}$ is the link delay. The phase noise $\phi_{rt}(t)$ accumulated by the light on the round-trip is

$$\phi_{rt}(t) = \int_{0}^{L} \left[ \delta \phi\left(z, t - 2\tau + \frac{z}{c_{n}}\right) + \delta \phi\left(z, t - \frac{z}{c_{n}}\right) \right]dz 
\approx \int_{0}^{L} 2\delta \phi(z, t - \tau)dz. \tag{2}$$

In Eq. (2), the approximation assumes a linear trend of the fiber perturbation at position $z$ between the forward
and the backward paths. This is justified for perturbations that act on timescales much longer than \( \tau \), which is the case of interest in most applications. Within this approximation, the phase noise in \( z = L \) is compensated by subtracting half of the round-trip signal phase from the forward signal phase:

\[
\varphi_{r,\text{comp}}(t) = \varphi_r(t) - \frac{1}{2} \varphi_n(t),
\]

where the factor 1/2 takes into account that the noise estimation is performed on a double pass in the fiber. From Eqs. (1) - (2), \( \varphi_{r,\text{comp}}(t) \) can be calculated as

\[
\varphi_{r,\text{comp}}(t) \approx \int_0^L \left[ \delta \varphi(z, t - \tau + \frac{z}{c_n}) - \delta \varphi(z, t - \tau) \right] dz \\
\approx \int_0^L \frac{z}{c_n} \frac{d}{dt} \delta \varphi(z, t) dz = \int_0^L h(z, t) \ast \delta \varphi(z, t) dz.
\]

(4)

In the second line of Eq. (4), the difference between \( \delta \varphi(z, t - \tau + \frac{z}{c_n}) \) and \( \delta \varphi(z, t - \tau) \) has been rewritten in terms of their time derivative. Again, this is justified for perturbations which act on timescales much longer than \( \tau \). In the last step, we introduced \( h(z, t) = \frac{z}{c_n} \delta \tilde{\varphi}(t) \), with \( \delta(t) \) the time derivative of the Dirac delta generalized function. \( h(z, t) \) is the impulse response which, for each \( z \), processes the local phase perturbation. Considering the contribution of each fiber segment with length \( dz \), we can apply the fundamental theorem of spectral analysis [16] that expresses the power spectrum of the output of a linear and time-invariant system in terms of the input power spectrum. In our case,

\[
S_{r,\text{comp}}(z,f) = |H(z,f)|^2 S_{\delta \varphi}(z,f),
\]

(5)

where \( S_{r,\text{comp}}(z,f) \) and \( S_{\delta \varphi}(z,f) \) are the noise spectral density contributions of a section \( dz \) of fiber at position \( z \) for the compensated and free fiber, respectively. \( |H(z,f)|^2 = (2\pi f z/c_n)^2 \) is the square modulus of the \( h(z,t) \) Fourier transform, i.e., the transfer function. If the noise is uncorrelated along the fiber, the noise contributions coming from different \( z \) positions are independent and sum up, so that we can write

\[
S_{r,\text{comp}}(f) = \int_0^L |H(z,f)|^2 S_{\delta \varphi}(z,f) dz.
\]

(6)

Assuming, in addition, that the fiber noise is uniformly distributed along the link, i.e., \( S_{\delta \varphi}(z,f) = S_r(f)/L \), an integration leads to

\[
S_{r,\text{comp}}(f) = \frac{1}{3} (2\pi f \tau)^2 S_r(f).
\]

(7)

where \( S_r(f) \) is the phase noise power spectrum of the fiber. Although this result has been obtained in a passive approach, where the round-trip fiber noise is measured and used to correct the forward signal, the same limitation is found for actively compensated links [3]. We point out that we do not assume that the power spectrum of \( \varphi(t) \) is the square modulus of its Fourier transforms. In our approach, Eq. (7) is deduced from the definition of power spectral density in terms of the Fourier transform of its autocorrelation, i.e., the formal definition in the case of random processes [16,17].

To improve the limit posed by Eq. (7), we reconsider Eq. (3) supposing that the subtraction can be performed between arbitrarily time-delayed samples, with delay \( \alpha \):

\[
\tilde{\varphi}_{r,\text{comp}}(t) = \varphi_r(t) - \frac{1}{2} \varphi_n(t + \alpha),
\]

(8)

where \( \tilde{\varphi}_{r,\text{comp}} \) is the corresponding compensated phase. With the same assumptions adopted previously, i.e., noise uncorrelated and uniformly distributed along the link, Eqs. (6) and (7) are modified into

\[
\tilde{S}_{r,\text{comp}}(f) = \int_0^L |\tilde{H}(z,f)|^2 S_{\delta \varphi}(z,f) dz \\
= \frac{1}{3} (2\pi f)^2 (\tau^2 - 3\alpha \tau + 3\alpha^2) S_r(f),
\]

(9)

where \( |\tilde{H}(z,f)|^2 = (2\pi f)^2 (z/c_n - \alpha)^2 \). The noise conversion is minimized for \( \alpha = \tau/2 \) leading to

\[
\tilde{S}_{r,\text{comp}}(f) = \frac{1}{12} (2\pi f \tau)^2 S_r(f).
\]

(10)

The remarkable result expressed by Eq. (10) is that performing the subtraction of time-shifted phase data, the limit shown in Eq. (7) is overcome by 6 dB. More specifically, the noise compensation is optimized if the round-trip signal is shifted by \( \tau/2 \). This of course can be done only by post-processing the phases \( \varphi_r \) and \( \varphi_n \) acquired at both sides of the fiber. The synchronization is not a stringent requirement: in a real case, timing at the level of \( \tau/10 \) is feasible and is wide enough for most applications [18]. This has been demonstrated in [19] and the same approach can be followed in this context.

To verify the theoretical prediction of Eq. (10), we implement the setup shown in Fig. 1. A narrow-linewidth laser is generated by frequency-locking a fiber laser to a high-linesee Fabry–Perot cavity at the 10–14 stability level at 1 s [20]. The laser is split into two parts: a small fraction of the optical power is used as a local oscillator, and the other part travels the optical link. This is based on a 47 km long optical fiber buried in the metropolitan area of Turin (Italy), with both ends in our laboratory. Accordingly, \( \tau = 235 \) for this loop. The fiber is implemented on a dense wavelength division multiplexed (DWDM) architecture where the channel 44 of the International Telecommunication Union grid is dedicated to this experiment, whereas channels 21 and 22 are occupied by the Internet traffic. More details about this fiber loop can be found in [21]. At the remote link end, the radiation is frequency shifted by 40 MHz by the acousto-optic modulator AOM to distinguish the round-trip signal from the stray reflections along the fiber. A fraction of the delivered radiation is extracted and compared to the original one on photodiode PD2; this enables direct measurement of the link performances, rejecting the laser noise contribution. The other part is reflected by
Fig. 1. Experimental setup. The ultrastable laser is split into two beams. One part is used as a local oscillator; the other part travels the link and, at the remote end, is frequency shifted by the acousto-optic modulator AOM, extracted and compared to the local oscillator on photodiode PD1. A part of the light is reflected by the Faraday mirror FM2 and travels the link in the backward direction; the round-trip light is compared to the local oscillator on photodiode PD2; C1-C4 represent optical couplers. The beat notes on PD1 and PD2 are tracked by two direct digital synthesizers (DDSs); the phase corrections $\phi_r$ and $\phi_c$ are also sent to a PC for measurement. The gray empty box represents a realignment term, which can be measured in the local laboratory by synchronously acquiring the effect of its phase modulation $\phi_c$ on the round-trip and on the forward signals, we obtain

$$
\phi_r(t) = \phi_r^c(t) - \phi_r(t - \tau),
$$

where we define $\phi_r^c(t)$ as a function of the signals in the closed loop:

$$
\phi_r^c(t) = \phi_r(t) - \frac{1}{2} \phi_r'(t + \frac{1}{2} \tau) - \tau - \phi_r(t).
$$

The superscript “cl” identifies the signals in the closed loop approach. According to Eq. (11), we rewrite Eq. (8) as a function of the signals in the closed loop:

$$
\tilde{\phi}_r(t) = \phi_r(t) - \frac{1}{2} \phi_r'(t + \frac{1}{2} \tau) = \phi_r^c(t) - r(t),
$$

where $r(t)$ is given by

$$
r(t) = \phi_c(t - \tau) + \frac{\phi_c^c(t + \frac{1}{2} \tau) - \phi_c(t + \frac{3}{2} \tau) - \phi_c(t + \frac{3}{2} \tau)}{2}.
$$

$r(t)$ represents a realignment term, which can be measured in the local laboratory by synchronously acquiring the phase-correction $\phi_c(t)$ imposed to AOM and the PD1 beat note phase when the controller is active. This term can then be applied to the phase measurement at the remote end to improve the comparison during post-processing. Thus, the frequency dissemination is performed at the classical limit; in addition, the phase-measurement can be further processed to obtain the 6 dB improvement.
In conclusion, we demonstrate from a theoretical and an experimental point of view that the classical limit in the phase-stabilization of coherent optical links can be overcome by time-shifting the phase correction applied to the forward signal. Without a loss of generality, this improvement is demonstrated in a passive approach, where there is no active cancellation of the fiber noise. Furthermore, we suggest how to improve the apparatus to perform two tasks at the same time: the delivery of an ultrastable frequency signal and the off-line processing to increase the resolution in a frequency comparison. The digital implementation ensures precise timing of the measurements, which is essential for the synchronous acquisition, and is reliable and easily adapted to perform different tasks. This is particularly interesting for the next generation of atomic clocks, which are expected to achieve the $10^{-18}$ level of relative frequency stability in 100 s of averaging time [10–12]. In addition, it may benefit other applications, such as the long-haul fiber frequency dissemination, paving the way for new measurements in atomic and fundamental physics.

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